Optimal fidelity of teleportation of coherent states and entanglement

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We study the Braunstein-Kimble protocol for the continuous variable teleportation of a coherent state. We determine lower and upper bounds for the optimal fidelity of teleportation, maximized over all local Gaussian operations for a given entanglement of the two-mode Gaussian state shared by the sender (Alice) and the receiver (Bob). We also determine the optimal local transformations at Alice and Bob sites and the corresponding maximum fidelity when one restricts to local trace-preserving Gaussian completely positive maps.

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I. INTRODUCTION

Quantum teleportation [1] is the transfer of an unknown quantum state from a sender (Alice) to a receiver (Bob) by means of the entanglement shared by the two parties and appropriate classical communication. Bob recovers an exact copy of the state teleported to him by Alice only if the quantum channel is the ideal maximally entangled state, which however, in the case of continuous variables (CV), is an unphysical infinitely squeezed state [2]. Nevertheless, by considering the finite quantum correlations between the quadratures in a two-mode squeezed state, Braunstein and Kimble [3] proposed a realistic protocol employing a beam splitter and homodyne measurements, which approaches perfect teleportation in the limit of infinite degree of squeezing. This protocol (and its various extensions) has been then implemented by various groups [4]. The success of a teleportation experiment is quantified by the *fidelity* which, in the case of a pure input state $|\psi_{in}\rangle$, is given by $\mathcal{F} = \langle\psi_{in}|\rho_{out}|\psi_{in}\rangle$, where ρ_{out} denotes the output state of the protocol, and coincides with the probability of finding the input state $|\psi_{in}\rangle$ at the output.

The relation between the fidelity of CV teleportation $\mathcal{F}$ and the entanglement shared by the distant parties is nontrivial and it has been investigated in various papers [5, 6]. In fact, entanglement is the key resource that allows to beat any classical strategy for transmitting quantum states. One can say that genuine quantum teleportation has been performed only if $\mathcal{F} > \mathcal{F}_{cl}$, where $\mathcal{F}_{cl}$ is the classical fidelity threshold achievable by two cheating parties who can perform arbitrary local operations and classical communication (LOCC) but are not able to share entanglement, nor to directly transmit quantum systems [5]. This means that $\mathcal{F} > \mathcal{F}_{cl}$ is a sufficient condition for entanglement between Alice and Bob; it is not a necessary condition because one can have lower-than-classical fidelities even using an entangled state. This is due to the fact that the Braunstein-Kimble protocol chooses specific combinations of quadratures, and therefore $\mathcal{F}$, differently from entanglement, is not a local symplectic invariant: if the protocol employs combinations of quadratures which are inappropriate for the given shared entangled state, the resulting $\mathcal{F}$ is very low. The difference between of teleportation fidelity and entanglement opens two problems: i) the determination of the classical fidelity threshold $\mathcal{F}_{cl}$ for a given class of CV input states; ii) the optimization of the teleportation fidelity, for the chosen class of input states, over all possible LOCC strategies. Determining $\mathcal{F}_{cl}$ is a nontrivial quantum estimation problem which has been solved only for few classes of states: i) input coherent states with completely unknown amplitude ($\mathcal{F}_{cl} = 1/2$) [5]; ii) pure squeezed states with zero displacement and completely unknown degree of squeezing at a given phase (see Ref. [7] which obtained $\mathcal{F}_{cl} = 0.81$ by considering LOCC strategies in which one is only allowed to prepare squeezed thermal states); iii) pure squeezed states with completely unknown displacement and orientation in phase space but fixed degree of squeezing s ($\mathcal{F}_{cl} = \sqrt{s}/(1+s)$) [8]. Here we restrict to input coherent states, which represent the basic resource for many quantum communication schemes [9, 10].

The improvement of the teleportation of coherent states by means of local operations and its relation with the entanglement of the shared entangled state has been already discussed in a number of papers [6, 11, 12, 13]. Ref. [11] showed that in some cases the fidelity of teleportation may be improved by local squeezing transformations, while Ref. [12] showed that in the case of a shared asymmetric mixed entangled resource, teleportation fidelity can be improved even by a local *noisy* operation. Ref. [13] then considered the class of local trace-preserving Gaussian completely positive (TGCP) maps (those performed by first adding ancillary systems in Gaussian states, then performing unitary Gaussian transformations on the whole system, and finally discarding the ancillas), and maximized the fidelity over this class of operations.

Ref. [13] confirmed that the best local TGCP map maybe a noisy one, i.e., that teleportation fidelity can be increased even by decreasing the entanglement and increasing the noise of the shared entangled state. Ref. [13], however, did not discuss the relationship between entanglement and the optimal fidelity $\mathcal{F}_{opt}$. Ref. [6] instead found this relationship, but only for a subclass of symmetric Gaussian entangled state shared by Alice and Bob: for this class it is $\mathcal{F}_{opt} = (1+\nu)^{-1}$, where ν is the low-

est symplectic eigenvalue of the partial transposed (PT) state. The parameter ν provides a quantitative characterization of CV entanglement, because the logarithmic negativity $E_{\mathcal{N}}$ is related to ν by $E_{\mathcal{N}} = \max[0, -\ln \nu]$ [14].

The problem of finding the optimal LOCC strategy is non-trivial because the set of operations that Alice and Bob can adopt is very large. In fact, apart from local TGCP maps, they can adopt two further options: i) use *non-trace preserving* Gaussian operations in which some ancillary mode is subject to Gaussian measurement, i.e., projected onto a Gaussian state, rather than discarded [24]; ii) use local non-Gaussian operations (either with measurement on ancillas or not), i.e., those involving interactions which are non-quadratic in the canonical coordinates. The first class of maps, together with TGCP maps, forms the most general class of Gaussian completely positive (GCP) operations, capable of preserving the Gaussian nature of the state shared by Alice and Bob. Non-Gaussian operations instead will transform the initial Gaussian bipartite state of Alice and Bob into a non-Gaussian one, and they can also increase the fidelity of teleportation in some cases [25].

In this paper we generalize in various directions the results of Refs. [6, 13]. We show that if Alice and Bob share a bipartite Gaussian state with a given ν and one restricts to local GCP maps which preserve such a Gaussian nature, the optimized fidelity always satisfies

$$\frac{1+\nu}{1+3\nu} \leq \mathcal{F}_{opt} \leq \frac{1}{1+\nu}. \tag{1}$$

We also show that the upper bound is reached iff Alice and Bob share a symmetric entangled state. Moreover we determine the optimal local transformations at Alice and Bob sites and the corresponding value of $\mathcal{F}_{opt}$ as a function of the symplectic invariants of the shared CV entangled state when one restricts to local TGCP maps.

The paper is organized as follows. In Sec. II we provide the basic definitions of the problem, in Sec. III we prove and discuss the lower and upper bounds for the optimal teleportation fidelity for a given shared entanglement. In Sec. IV we discuss the properties of the optimal local map and derive its explicit form in the case of TGCP maps. Sec. V is for concluding remarks.

II. DEFINITION OF THE PROBLEM

The protocol for a perfect CV quantum teleportation based on ideal Einstein-Podolski-Rosen (EPR) correlations has been introduced by Vaidman [2], and then adapted to the finite correlations of a two-mode squeezed states by Braunstein and Kimble [3]. The idea can be easily shown in the Heisenberg picture. We consider two CV systems, each described by a pair of conjugate dimensionless quadratures $\hat{x}_k$ and $\hat{p}_k$ ($k = a, b$). Introducing the vector $\hat{\xi}^T \equiv (\hat{x}_a, \hat{p}_a, \hat{x}_b, \hat{p}_b)$, we can write the canonical commutation relations as $[\hat{\xi}_l, \hat{\xi}_m] = i\mathcal{J}_{lm}$ ($l, m = 1, ...4$), where

$$\mathcal{J} \equiv J \oplus J, \; J \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2}$$

and $\oplus$ denotes the usual direct sum operation.

Alice and Bob share a bipartite CV state with EPR-like correlations, i.e., a state which can be considered as an approximate simultaneous eigenstate of the combinations of quadratures $\hat{x}_a + \hat{x}_b$ and $\hat{p}_a - \hat{p}_b$, so that the variances of these two combinations are both close to zero. Alice also possesses an unknown input state with quadratures $\hat{x}_{in}, \hat{p}_{in}$ which she wants to teleport to Bob. Alice mixes the input mode with her part of the entangled state via a balanced beam splitter and she carries out a homodyne detection on each output mode, thereby measuring two commuting quadratures $\hat{x}_+ = (\hat{x}_a + \hat{x}_{in})/\sqrt{2}$ and $\hat{p}_- = (\hat{p}_{in} - \hat{p}_a)/\sqrt{2}$. After receiving the measured values x_+ and p_- from Alice, Bob uses this transmitted classical information to perform a suitable conditional displacement on his own mode, $\hat{x}_b \longrightarrow \hat{x}'_b \equiv \hat{x}_b + \sqrt{2}x_+$, $\hat{p}_b \longrightarrow \hat{p}'_b \equiv \hat{p}_b + \sqrt{2}p_-$.

If we assume ideal homodyne detectors on Alice site, and that the shared bipartite state is undisplaced (i.e., all mean values of Alice and Bob quadratures vanish), the EPR-like correlations $\hat{x}_a \simeq -\hat{x}_b$ and $\hat{p}_a \simeq \hat{p}_b$, together with Bob displacements, imply $\hat{x}'_b \simeq \hat{x}_{in}$ and $\hat{p}'_b \simeq \hat{p}_{in}$, i.e., Bob mode is described by a pair of conjugate variables very close to those of the input mode. In the Schrödinger picture, this is equivalent to teleport the input state to Bob with a fidelity very close to one.

We restrict to the case when the state shared by Alice and Bob ρ_{ab} is Gaussian, where a compact expression of the resulting fidelity of teleportation can be derived [13, 15, 16]. A bipartite CV state ρ_{ab} is Gaussian if its Wigner characteristic function $\Phi_{ab}(\vec{\xi}) \equiv \mathrm{Tr}[\rho_{ab} \exp(-i\vec{\xi}^T \cdot \hat{\xi})]$ (where $\vec{\xi}^T = (x_a, p_a, x_b, p_b)$ is the vector of phase-space variables corresponding to $\hat{\xi}^T$), is Gaussian, i.e., $\Phi_{ab}(\vec{\xi}) = \exp(-\vec{\xi}^T V \vec{\xi}/4 + i\vec{d}^T\vec{\xi})$. We have assumed that Alice and Bob share a zero-displacement state, implying $\vec{d} = 0$. Therefore ρ_{ab} is fully characterized only by its correlation matrix (CM) V, whose generic element is defined as $V_{lm} \equiv \langle \Delta\hat{\xi}_l \Delta\hat{\xi}_m + \Delta\hat{\xi}_m \Delta\hat{\xi}_l \rangle$ where $\Delta\hat{\xi}_l \equiv \hat{\xi}_l - \langle \hat{\xi}_l \rangle$. The CM satisfies the uncertainty principle $V + i\mathcal{J} \geq 0$ [17], and can always be put in the block form

$$V \equiv \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \tag{3}$$

where A, B, and C are 2×2 real matrices. Using characteristic functions, it is straightforward to prove [13, 15] that, if the input state is a single-mode Gaussian state with CM V_{in}, the fidelity of teleportation is given by

$$\mathcal{F} = \frac{2}{\sqrt{\det(2V_{in} + N)}}, \tag{4}$$

where

$$N = ZAZ + ZC + C^T Z + B, \tag{5}$$

with $Z = \mathrm{diag}(1, -1)$ [18]. The 2×2 matrix N is semi-positive definite, $N \geq 0$, it describes the noise added to the teleported state, and it is equal to zero only in the ideal situation of perfect EPR correlations between Alice and Bob. As discussed in the introduction, we shall restrict to the case of input coherent states, $V_{in} = I$, so that Eq. (4) reduces to

$$\mathcal{F} = \frac{2}{\sqrt{4 + 2\mathrm{Tr}N + \det N}}. \qquad (6)$$

The problem afforded in this paper, i.e., the maximization of the teleportation fidelity over all possible Gaussian LOCC strategies for a given Alice-Bob entanglement, therefore means to determine the optimal local transformation of matrices A, B and C which makes N as small as possible.

As showed in [16], using an unbalanced beam splitter is equivalent, for the teleportation protocol, to a squeezing operation by Alice. Therefore the optimization over all Alice and Bob local operations includes also any eventual modification of the beam splitter used for the joint homodyne measurement.

III. UPPER AND LOWER BOUNDS FOR THE FIDELITY OF TELEPORTATION

In this section we prove Eq. (1), i.e., the upper and lower bounds for the fidelity of teleportation for input coherent states. An important preliminary result enabling us to derive the two bounds is the fact that the optimal noise matrix N is very simple: in fact, the maximum teleportation fidelity is obtained when N is proportional to the 2×2 identity matrix I. More precisely, we have the following

Lemma 1 (**Optimal noise matrix**). *If ω_{opt} is an optimal local GCP map which gives the maximum of the fidelity $\mathcal{F}_{max}$ for the teleportation of a coherent state, then the resulting noise matrix is a multiple of the identity, that is $N_{opt} = 2n_{opt}I$.*

Proof. First of all we observe that for a 2×2, symmetric and positive semidefinite matrix like N, the condition $N = 2n_{opt}I$ is equivalent to $\mathrm{Tr}N = 2\sqrt{\det N}$. Therefore we have to show that ω_{opt} is such that $\mathrm{Tr}N_{opt} = 2\sqrt{\det N_{opt}}$. We do this by reductio ad absurdum supposing that ω_{opt} gives a noise matrix with $\mathrm{Tr}N_{opt} > 2\sqrt{\det N_{opt}}$. However, within the class of local GCP maps, there exists a subclass of local symplectic (i.e., unitary Gaussian) maps realized by a generic symplectic S_b on Bob mode, and the associated symplectic map $S_a = ZS_bZ$ on Alice mode, which act as an *effective* symplectic transformation on N_{opt}, $N'_{opt} = S_b N_{opt} S_b^T$ (see Eq. (5)). We can always choose S_b such that $N'_{opt} = \sqrt{\det N_{opt}} I$, for which $\mathrm{Tr}N'_{opt} = 2\sqrt{\det N_{opt}} < \mathrm{Tr}N_{opt}$ while $\det N'_{opt} = \det N_{opt}$. However, we see from Eq. (6), that this local symplectic operation *increases* the teleportation fidelity, but this is absurd because we assumed from the beginning that N_{opt} is optimal. □

From this lemma and Eq. (6) we can therefore rewrite the optimal fidelity of teleportation in terms of the single positive parameter $n_{opt} = \sqrt{\det N_{opt}}/2$ as

$$\mathcal{F}_{opt} = \frac{1}{1 + n_{opt}}. \qquad (7)$$

We can now derive an upper bound for $\mathcal{F}_{opt}$ for a given entanglement of the state shared by Alice and Bob. We quantify such entanglement in terms of the lowest partially transposed (PT) symplectic eigenvalue, ν. Such a parameter cannot be improved (i.e., decreased) by local operations and therefore provides a quantitative characterization of CV entanglement [14]. It is a local symplectic invariant and it can be expressed in terms of the four local symplectic invariants $\det A$, $\det B$, $\det C$ and $\det V$ as

$$\nu = 2^{-1/2} \left[\Sigma(V) - \left(\Sigma(V)^2 - 4 \det V \right)^{1/2} \right]^{1/2}, \qquad (8)$$

where $\Sigma(V) \equiv \det A + \det B - 2 \det C$.

Theorem 1 (**Upper bound**). *For a given Gaussian bipartite state shared by Alice and Bob, with lowest PT symplectic eigenvalue ν, the fidelity of the teleportation of a coherent state is limited from above by*

$$\mathcal{F}_{opt} \leq \frac{1}{1 + \nu}. \qquad (9)$$

Proof. Let us suppose that we can achieve a larger fidelity $\mathcal{F} = 1/(1+n_{opt})$ with $0 < n_{opt} < \nu$. Alice can in principle have at her disposal a two-mode squeezed state, with the usual correlation matrix

$$W = \begin{pmatrix} I \cosh r & -Z \sinh r \\ -Z \sinh r & I \cosh r \end{pmatrix}, \qquad (10)$$

(r is the squeezing parameter) and use this two-mode squeezed state, together with the bipartite state shared with Bob already optimized over all local GCP maps, to implement a CV entanglement swapping protocol [19]. In fact, by mixing at a balanced beam splitter her mode of the bipartite state shared with Bob and one part of the two-mode squeezed state, and performing homodyne measurements at the output, Bob mode gets entangled with the remaining part of the two-mode squeezed state in Alice hands. Since the noise added to the teleported state is $N_{opt} = 2n_{opt}I$, it is straightforward to see that the two remaining modes are then described by the following CM

$$W_{swap} = \begin{pmatrix} I \cosh r & -Z \sinh r \\ -Z \sinh r & I[2n_{opt} + \cosh r] \end{pmatrix}. \qquad (11)$$

In other words, before entanglement swapping, Alice and Bob shared an entangled state with CM V and entanglement characterized by ν; after entanglement swapping,

they share a state with CM W_{swap}. In the limit of infinite squeezing the lowest PT symplectic eigenvalue of W_{swap} tends to n_{opt}, i.e., $\lim_{r\to\infty}\nu_{swap} = n_{opt}$. Since we supposed $n_{opt} < \nu$, this means that for a sufficiently large squeezing parameter r, $\nu_{swap} < \nu$, i.e., Alice and Bob have increased their entanglement. However this is impossible because we have employed only local operations [20]. Therefore it must be $n_{opt} \geq \nu$. □

We complete the characterization of the optimal fidelity of teleportation in terms of the entanglement shared by the two distant parties by providing also a lower bound for $\mathcal{F}_{opt}$, proving in this way the result of Eq. (1).

Theorem 2 (**Lower bound**). *For a given Gaussian bipartite state shared by Alice and Bob, with lowest PT symplectic eigenvalue ν, the fidelity of the teleportation of a coherent state is limited from below by*

$$\mathcal{F}_{opt} \geq \frac{1+\nu}{1+3\nu}. \tag{12}$$

Proof. From the definition of symplectic eigenvalue, one has that a 4×4 symplectic matrix S exists which diagonalizes $\Lambda V \Lambda$ ($\Lambda = \mathrm{diag}(Z, I)$), i.e., the PT matrix of the CM V. This means $S\Lambda V\Lambda S^T = \mathrm{diag}(\nu,\nu,\mu,\mu)$, where μ is the largest PT symplectic eigenvalue. By writing S in 2×2 block form

$$S = \begin{pmatrix} W_a & W_b \\ W_c & W_d \end{pmatrix}, \tag{13}$$

and rewriting the diagonalization condition for the upper 2×2 block only, one gets the following condition

$$W_a Z A Z W_a^T + W_a Z C W_b^T + W_b C^T Z W_a^T + W_b B W_b^T = \nu I. \tag{14}$$

The symplectic transformation S transforms the vector of quadratures $\hat{\xi}$ into $\hat{\xi}' = (x_a', y_a', x_b', y_b')^T = S\hat{\xi}$ and the PT vector into $\hat{\xi}'' = (x_a'', y_a'', x_b'', y_b'')^T = S\Lambda\hat{\xi}$. One has $[x_a', y_a'] = i$, because commutation relation are preserved by S, implying

$$\det W_a + \det W_b = 1. \tag{15}$$

The commutation relation is instead not preserved for the PT transformed quadratures, and introducing a real parameter ϵ such that $[x_a'', y_a''] = i\epsilon$, we get another condition for the two upper blocks of S,

$$-\det W_a + \det W_b = \epsilon, \tag{16}$$

which together with Eq. (15), gives the parametrization

$$\det W_a = (1-\epsilon)/2, \qquad \det W_b = (1+\epsilon)/2. \tag{17}$$

Now, since $\Delta x_a''^2 = \Delta y_a''^2 = \nu/2$, the Heisenberg uncertainty principle imposes that $|\epsilon| \leq \nu$ and in particular for every entangled state we have $|\epsilon| \leq \nu < 1$. This latter condition, together with Eq. (17), suggests an alternative parametrization in terms of the angle $\theta = \arctan\sqrt{(1-\epsilon)/(1+\epsilon)}$ $(0 < \theta < \pi/2)$,

$$\sqrt{\det W_a} = \sin\theta, \quad \sqrt{\det W_b} = \cos\theta. \tag{18}$$

The 2×2 matrices W_a and W_b and the parameter θ allow to construct an appropriate local map which will lead us to derive a lower bound for the fidelity. This local map is a TGCP map which, at the level of CM, acts as [21, 22, 23]

$$V \to V' = SVS^T + G, \tag{19}$$

with S and G satisfying

$$G + i\mathcal{J} - iS\mathcal{J}S^T \geq 0. \tag{20}$$

If the TGCP map is local, then $S = S_a \oplus S_b$ and $G = G_a \oplus G_b$, with $G_k + iJ - iS_k J S_k^T \geq 0$ $(k = a, b)$.

The desired local TGCP map ω_θ is defined in terms of S_a, S_b, G_a and G_b in the following way

$$S_a = \begin{cases} ZW_aZ\,[\cos\theta]^{-1} & 0<\theta\leq\pi/4 \\ ZW_aZ\,[\sin\theta]^{-1} & \pi/4\leq\theta<\pi/2, \end{cases} \tag{21a}$$

$$S_b = \begin{cases} W_b\,[\cos\theta]^{-1} & 0<\theta\leq\pi/4 \\ W_b\,[\sin\theta]^{-1} & \pi/4\leq\theta<\pi/2, \end{cases} \tag{21b}$$

$$G_a = \begin{cases} \left[1-\tan^2\theta\right] I & 0<\theta\leq\pi/4 \\ 0 & \pi/4\leq\theta<\pi/2, \end{cases} \tag{21c}$$

$$G_b = \begin{cases} 0 & 0<\theta\leq\pi/4 \\ \left[1-\cot^2\theta\right] I & \pi/4\leq\theta<\pi/2. \end{cases} \tag{21d}$$

By applying Eqs. (5), (14) and (19), one can see that this local TGCP map transforms the noise matrix N into a final matrix proportional to the identity, given by

$$N = [\nu/\cos^2\theta + 1 - \tan^2\theta]I, \quad 0<\theta\leq\pi/4, \tag{22}$$

$$N = [\nu/\sin^2\theta + 1 - \cot^2\theta]I, \quad \pi/4\leq\theta<\pi/2. \tag{23}$$

It is however convenient to come back to the parametrization in terms of ϵ, which allows to express the final N in a unique way, for $0<\theta<\pi/2$. In fact, from Eqs. (22)-(23), one gets

$$N = 2\frac{\nu+|\epsilon|}{1+|\epsilon|}I, \tag{24}$$

which, inserted into Eq. (7), yields

$$\mathcal{F} = \frac{1+|\epsilon|}{1+\nu+2|\epsilon|}. \tag{25}$$

From the condition imposed by the Heisenberg uncertainty principle $0 \leq |\epsilon| \leq \nu$, we see that the fidelity is minimum when $|\epsilon| = \nu$, so that we get the following lower bound

$$\mathcal{F}_{opt} \geq \frac{1+\nu}{1+3\nu}. \tag{26}$$

□

Theorems 1 and 2 provide a very useful characterization of the optimal fidelity which can be achieved with Gaussian local operations at Alice and Bob site. In fact, the bounds are quite tight because the region between the upper and the lower bound is quite small (see Fig. 1). Therefore, by simply computing the lowest PT symplectic eigenvalue of the CM of the shared state and using the bounds, one gets a good estimate of the maximum fidelity that can be obtained with appropriate local operations. In fact, the error provided by the bounds is never larger than 0.086 (see Fig. 2).

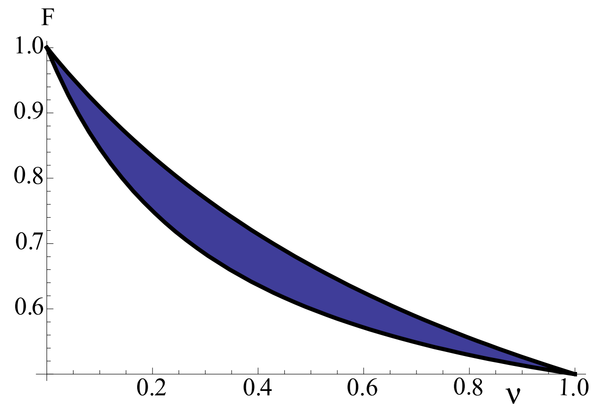

FIG. 1: (Color online) Plot of the upper and lower bounds (Eq. (9) and (12) respectively) for the fidelity of teleportation of coherent states. The blue region is the allowed region in the $(\mathcal{F}, \nu)$ plane.

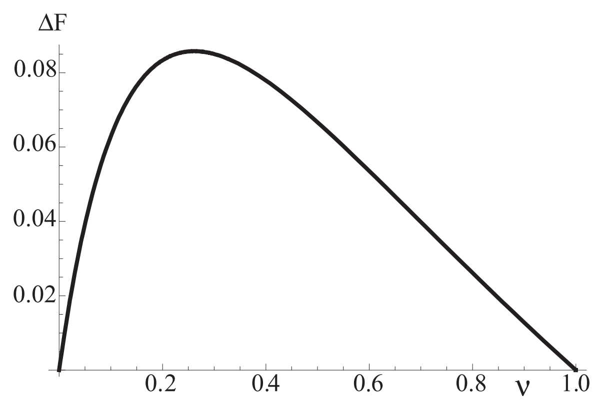

FIG. 2: Plot of the distance between the upper and lower bounds for the teleportation fidelity versus the allowed values of ν for an entangled state between Alice and Bob. We see that the error $\Delta\mathcal{F}$ under which we can estimate the maximum fidelity is less than 0.086.

Corollary 1 (Upper bound achieved in the symmetric case). *The upper bound $\mathcal{F}_{opt} = 1/(1+\nu)$ is achieved iff the bipartite Gaussian state shared by Alice and Bob is symmetric. The optimal local transformation in the symmetric case is a local symplectic map.*

Proof. The "if" part of the theorem directly follows as a special case of the preceding proof. If the Gaussian state shared by Alice and Bob is symmetric, it is $\det W_a = \det W_b$, implying $\epsilon = 0$. Then, Eq. (25) shows that in this case the fidelity reaches the upper bound, $\mathcal{F}_{opt} = 1/(1+\nu)$. Moreover in this case $\theta = \pi/4$ and the local TGCP map of Eqs. (21) is optimal and it is a symplectic one, with $S_a = ZW_aZ\sqrt{2}$, $S_b = W_b\sqrt{2}$, $G_a = G_b = 0$. The "only if" part instead can be easily proved by using the result of Theorem 3 about the CM of the optimized bipartite state shown in the following section. The proof is given in the Appendix. □

This latter corollary provides the generalization of the result of Ref. [6], which obtained the same relation between optimal fidelity and ν but by considering only a special class of symmetric Gaussian bipartite state for Alice and Bob, obtained by mixing at a beam splitter two single-mode thermal squeezed states.

IV. DETERMINATION OF THE OPTIMAL LOCAL MAP

We have derived a lower bound for the optimal fidelity of teleportation of coherent states, by explicitly constructing the family of local TGCP maps ω_θ of Eq. (21), which transform Alice and Bob shared state so that the corresponding fidelity of teleportation is given by Eq. (25), interpolating between the lower and upper bound of Theorems 1 and 2 by varying $\epsilon = \cos 2\theta$. The map ω_θ is symplectic only for $\theta = \pi/4$ and in this case it is the local optimal map for a symmetric shared state, since it reaches the upper bound of Theorem 1 (see Corollary 1). When $\theta \neq \pi/4$, ω_θ is a noisy (i.e., non unitary) map, and it is not optimal in general, because one cannot exclude that different LOCC strategies by Alice and Bob may yield a better noise matrix N and therefore a larger value of the teleportation fidelity. As discussed in the introduction, here we shall study the optimization of the teleportation by restricting to GCP maps, which preserves the Gaussian nature of the bipartite state initially shared by Alice and Bob.

In this section we shall derive two results: i) the general form of the final CM of the bipartite Gaussian state after the optimization over all local GCP maps; ii) the optimal local TGCP map, i.e., the local TGCP map which maximizes the teleportation fidelity when one restricts to TGCP maps only, excluding in this way measurements of ancillary modes.

Ref. [13] already provided the analytical procedure for the determination of all the parameters of the optimal TGCP map. Here, by further elaborating the approach of Ref. [13], we will show that the optimal TGCP map can always be written in a simple form, as a local symplectic operation eventually followed by a single mode *attenuation* [26], either at Alice or Bob site.

A. Standard form of the correlation matrix of the optimized bipartite state

In this subsection we show that, even if we do not know the specific form of the optimal GCP map, one can always characterize it indirectly by determining the general form of its outcome, i.e., the general form of the CM of the final Gaussian state shared by Alice and Bob after the maximization. We begin with the following lemma.

Lemma 2 (**Standard form III**). *The correlation matrix V of every bipartite Gaussian state can be transformed by local symplectic operations into the following normal form*

$$V_1 = \begin{pmatrix} n_1 & & -d_1 & \\ & n_2 & & d_2 \\ -d_1 & & m_1 & \\ & d_2 & & m_2 \end{pmatrix}, \tag{27}$$

where all the coefficients are positive and satisfy the following constraints

$$n_1 - n_2 = m_1 - m_2 = d_1 - d_2 = \lambda, \qquad \lambda \in \mathbb{R}. \tag{28}$$

That is:

$$V_1 = \begin{pmatrix} n+\lambda & & -d-\lambda & \\ & n & & d \\ -d-\lambda & & m+\lambda & \\ & d & & m \end{pmatrix}. \tag{29}$$

Proof. It is well known that it is possible to transform every V of an entangled state in the usual normal form (standard form I) [27, 28]

$$V_N = \begin{pmatrix} a & & -c_1 & \\ & a & & c_2 \\ -c_1 & & b & \\ & c_2 & & b \end{pmatrix}, \tag{30}$$

where all the coefficients are positive. Now we perform a local symplectic operation composed by two local squeezing operations, $S_a = \mathrm{diag}(\sqrt{r_a}, 1/\sqrt{r_a})$ and $S_b = \mathrm{diag}(\sqrt{r_b}, 1/\sqrt{r_b})$. We impose the first two conditions $n_1 - n_2 = m_1 - m_2 = \lambda$,

$$a(r_a - r_a^{-1}) = b(r_b - r_b^{-1}) = \lambda, \tag{31}$$

which solved for positive r_a and r_b give

$$r_a(\lambda) = \lambda/(2a) + \sqrt{1 + (\lambda/2a)^2}, \tag{32}$$
$$r_b(\lambda) = \lambda/(2b) + \sqrt{1 + (\lambda/2b)^2}. \tag{33}$$

Now we impose the last constraint $d_1 - d_2 = \lambda$, that is

$$c_1\sqrt{r_a(\lambda) r_b(\lambda)} - c_2/\sqrt{r_a(\lambda) r_b(\lambda)} = \lambda. \tag{34}$$

Our lemma is proved if there is at least one solution λ of Eq. (34). Since $\lambda = 0$ is the trivial solution when $c_1 = c_2$ and $V_N = V_1$, we can exclude this particular case and divide Eq. (34) by λ. Therefore we have to show that the equation

$$f(\lambda) = \frac{1}{\lambda}[c_1\sqrt{r_a(\lambda) r_b(\lambda)} - c_2/\sqrt{r_a(\lambda) r_b(\lambda)}] = 1 \tag{35}$$

admits at least one real solution. If $|\lambda| \gg a$ and $|\lambda| \gg b$, we can power expand the square roots in Eqs. (32)-(33) so that we easily find the following limits for $f(\lambda)$:

$$\lim_{\lambda\to\infty} f(\lambda) = c_1/\sqrt{ab} \le 1, \tag{36}$$
$$\lim_{\lambda\to-\infty} f(\lambda) = c_2/\sqrt{ab} \le 1, \tag{37}$$
$$\lim_{\lambda\to 0^\pm} f(\lambda) = \pm\mathrm{sign}(c_1 - c_2)\infty. \tag{38}$$

The inequalities in Eqs. (36)-(37) follow from the Cauchy-Schwartz inequality $\langle x_1^2\rangle\langle x_2^2\rangle \ge \langle x_1 x_2\rangle^2$ applied to the quadrature operators of the two modes. Given the three limits (36), (37) and (38), since $f(\lambda)$ is continuous everywhere except at the origin, at least one solution of Eq. (35) exists. Moreover this solution has the same sign of $c_1 - c_2$. □

We have defined the standard form of lemma 2 as standard form III because it is very similar to the standard form II defined in Ref. [27] for the determination of a necessary and sufficient entanglement criterion for bipartite Gaussian states. In particular the two standard forms coincide in the special case of a symmetric bipartite state ($n_1 = m_1$ and $n_2 = m_2$ or equivalently $n = m$).

Theorem 3 (**Form of the CM of the optimized bipartite state**). *The optimal GCP map ω_{opt} maximizing the teleportation fidelity is such that the CM of the transformed bipartite state is in the standard form III V_1 defined by Eqs. (27)-(29).*

Proof. By means of local symplectic operations, we can always put the CM of the bipartite state of Alice and Bob in the form of Eq. (27), but without the constraints of Eq. (28). We first restrict to local symplectic operations and show that the optimal local symplectic operation always transforms to a state with a CM satisfying the constraints of Eq. (28). Since the CM is tridiagonal, then any possible optimal map must be a squeezing transformations of the two modes given by $S_a = \mathrm{diag}(r_a, r_a^{-1})$ and $S_b = \mathrm{diag}(r_b, r_b^{-1})$ [13]. Let us define

$$\alpha(r_a, r_b) = r_a^2 n_1 - 2d_1 r_a r_b + r_b^2 m_1, \tag{39}$$
$$\beta(r_a, r_b) = r_a^{-2} n_2 - 2d_2 (r_a r_b)^{-1} + r_b^{-2} m_2, \tag{40}$$

so that the noise matrix of Eq. (5) is equal to $N = \mathrm{diag}(\alpha, \beta)$. The optimal map must minimize $\det N = \alpha\beta$, and therefore we impose that

$$\nabla \alpha(r_a, r_b)\beta(r_a, r_b) = 0, \tag{41}$$

where $\nabla = (\partial_{r_a}, \partial_{r_b})$. Due to Lemma 1, we must also have that $\alpha(r_a, r_b) = \beta(r_a, r_b) \neq 0$, and therefore Eq. (41) reduces to

$$\nabla[\alpha(r_a, r_b) + \beta(r_a, r_b)] = 0. \tag{42}$$

The CM of the transformed state is the optimized one iff the optimal local symplectic operation is the identity map, that is, if $\alpha(1,1) = \beta(1,1)$ and

$$\nabla[\alpha(r_a, r_b) + \beta(r_a, r_b)]\Big|_{r_a=r_b=1} = 0. \tag{43}$$

It is easy to check that these conditions are satisfied iff

$$n_1 - n_2 = m_1 - m_2 = d_1 - d_2, \tag{44}$$

which are exactly the constraints of Eq. (28). The theorem is proved if we show that the normal form V_1 of Eq. (29) is actually kept also if one maximizes over the broader class of GCP maps. In fact, if by reductio ad absurdum, we assume that an optimal (non-symplectic) GCP map exists leading to a CM not satisfying the constraints of Eq. (28), we could always apply a further symplectic map which, by repeating the maximization above, would transform to a state with a CM satisfying the constraints (28) and yielding a larger teleportation fidelity. But this is impossible, because it contradicts the initial assumption of starting from the optimal bipartite state. □

In other words, the CM of the state shared by Alice and Bob after the maximization of the teleportation fidelity is the one with the standard form III of Eq. (29) because it is the unique CM for which the optimal map is the identity operation on both Alice and Bob site.

B. Optimal trace-preserving Gaussian CP map

In the former subsection we have determined the form of the CM of the optimized state of Alice and Bob, without determining however which is the local GCP map which maximizes the teleportation fidelity. Here we find this optimal map, restricting however to the smaller class of *trace-preserving* GCP maps. The case of Gaussian maps including Gaussian measurements on ancillas will be afforded elsewhere.

Ref. [22] has introduced the notion of *minimal noise* TCGP maps, as the extremal solution of the condition of Eq. (20). These maps are the ones that, for a given matrix S, possess the "smallest" positive matrix G realizing a CP map. It is easy to check that a minimal noise TGCP map satisfies the relation $\det G = (1 - \det S)^2$. An example of minimal noise TGCP map is an *attenuation* [26], i.e., the transmission of a single boson mode through a beam splitter with transmissivity τ $(0 \le \tau \le 1)$, such that

$$a \to \tau a + \sqrt{1-\tau^2}\, a_V, \tag{45}$$

where $a = (\hat{x} + i\hat{p})/\sqrt{2}$ is the annihilation operator of the mode, and a_V that of the vacuum mode entering the unused port of the beam splitter.

It is evident that the TGCP map maximizing the teleportation fidelity has to be a minimal noise TGCP map [13]. We prove now a useful decomposition theorem.

Theorem 4 (Decomposition of TGCP maps). *A minimal noise TGCP map ω on a single mode system with $\det G \le 1$ can always be decomposed into a symplectic transformation σ_1, followed by an attenuation τ and by a second symplectic transformation σ_2, that is,*

$$\omega = \sigma_2 \circ \tau \circ \sigma_1. \tag{46}$$

Therefore a local minimal noise TGCP map on a bipartite CV system can always be decomposed into a local symplectic map, followed by the tensor product of two local attenuations and by a second local symplectic map.

Proof. We consider a generic minimal noise TGCP map such that $V \to V' = SVS^T + G$ for a generic CM V, with $\det G = (1 - \det S)^2$. G is a positive symmetric matrix, and therefore a symplectic matrix T_2 exists such that $G = T_2 T_2^T (1 - s)$, where $s = \det S$. We then define the symplectic matrix $T_1 = \frac{1}{\sqrt{s}} T_2^{-1} S$, and we also consider an attenuation map with transmissivity $\sqrt{s}$. If we now first apply the symplectic map defined by T_1, then the attenuation map and finally the second symplectic map defined by T_2, by using the relations $T_2 \sqrt{s} T_1 = S$ and $G = T_2 T_2^T (1 - s)$, one can check that the composition of the three maps reproduces the given TGCP map. □

Corollary 2. *A minimal noise TGCP map ω on a single mode system with G proportional to the identity matrix, i.e., $G = (1 - s)I$ $(0 \le s \le 1)$ can always be decomposed into a symplectic transformation σ_1, followed by an attenuation τ,*

$$\omega = \tau \circ \sigma_1. \tag{47}$$

Therefore a local minimal noise TGCP map on a bipartite CV system with $G_i = (1 - s_i)I$ $(0 \le s_i \le 1,\ i = a, b)$ can always be decomposed into a local symplectic map, followed by the tensor product of two local attenuations.

Proof. It is sufficient to repeat the former proof and consider that, since $G = (1 - s)I$, $T_2 = I$ and therefore the second symplectic map is the identity operation. □

This latter case is of interest because the optimal TGCP map must have in fact the property $G_i = (1-s_i)I$, $i = a, b$. To show this we first simplify the scenario by exploiting the results of Ref. [13], which provides the general analytical procedure to derive the optimal local TGCP map. In fact, Ref. [13] shows that when the optimal local TGCP map is a minimal noise, non-symplectic one, it can be performed on one site only, i.e., either on Alice or on Bob alone. Suppose that the non-symplectic map is performed on Bob site; it is straightforward to see that, under a generic local TGCP map $V \to V' = SVS^T + G$, with $S = I \oplus S_b$ and $G = I \oplus G_b$, the noise matrix N transforms according to $N \to N' = \Gamma + G_b$ where

$$\Gamma = ZAZ + ZCS_b^T + S_b C^T Z + S_b B S_b^T. \tag{48}$$

Ref. [13] shows that the optimal map is such that $\Gamma \propto G_b$, and since Lemma 1 shows that the optimal N is proportional to the identity, this implies that G_b must be proportional to the identity. Therefore Corollary 2 leads us to conclude that *the optimal local TGCP map is either a local symplectic map, or a local symplectic map followed by an attenuation by a beam splitter, placed either on Alice or on Bob mode.*

Theorem 4 and Corollary 2 therefore provide a very simple and clear description of the TGCP map which maximizes the teleportation fidelity, which is not evident in the treatment of Ref. [13]. We can further characterize the optimal local TGCP map by determining: i) the form of the first local symplectic map; ii) the conditions under which the optimal local operation is noisy, i.e., when one has also to add a beam splitter with appropriate transmissivity on Alice or Bob mode in order to maximize the teleportation fidelity. In order to do that we first need a further lemma, similar to lemma 2.

Lemma 3. *For any given positive real parameter η, the correlation matrix V of every bipartite Gaussian state can be transformed by local symplectic operations into the following normal form*

$$V_\eta = \begin{pmatrix} n_1 & & -d_1 & \\ & n_2 & & d_2 \\ -d_1 & & m_1 & \\ & d_2 & & m_2 \end{pmatrix}, \tag{49}$$

where all the coefficients are positive and satisfy the following constraint

$$n_1 - n_2 = \eta(d_1 - d_2) = \eta^2(m_1 - m_2) = \lambda, \qquad \lambda \in \mathbb{R}. \tag{50}$$

That is, we have a family of normal forms depending on the parameter η,

$$V_\eta = \begin{pmatrix} n+\lambda & & -d-\lambda/\eta & \\ & n & & d \\ -d-\lambda/\eta & & m+\lambda/\eta^2 & \\ & d & & m \end{pmatrix}. \tag{51}$$

Proof. With the same procedure used in the proof of Lemma 2, we arrive to an equation similar to (35), while the corresponding three limits are exactly the same of (36), (37) and (38), since the factor η cancels out. As a consequence the continuity argument is valid also in this case, and therefore, for every fixed parameter η, one can find a transformation which puts the CM in the normal form V_η. □

We notice two facts that will be useful for the next theorem: i) the optimal CM standard form of Theorem 3, V_1, belongs to the class of normal forms V_η, since it is obtained for $\eta = 1$; ii) when $0 < \eta < 1$, the state with CM V_η is transformed into the Gaussian state with CM equal to V_1 when a beam splitter with transmissivity η is put on Bob mode. We then arrive at the theorem about the optimal TGCP map.

Theorem 5. *The optimal local TGCP map maximizing the fidelity of teleportation of coherent states can always be decomposed into a local symplectic map, eventually followed by an attenuation either on Alice or on Bob mode. The first local symplectic map is the one transforming the CM of the Gaussian state shared by Alice and Bob into one particular normal form of the family V_η defined in Eqs. (49)-(50), with $0 < \eta \le 1$. One has to add the attenuation on one of the two modes for realizing the optimal TGCP map if there is a value of η, let us say $\eta = \tau$, such that the coefficients of V_τ satisfy the relations*

$$\tau = \frac{d}{m-1}, \qquad \tau < 1. \tag{52}$$

If instead the condition of Eq. (52) is never satisfied in the interval $\eta \in [0, 1)$, the optimal TGCP map is formed only by the local symplectic map transforming the CM into the normal form V_1 (i.e., $\eta = 1$ and no attenuation is required).

Proof. Using Lemma 3, we can always apply a local symplectic map which transform the CM into one of the form of the family of Eq. (51), with $\eta = \tau$. We then apply an attenuation on Bob mode with transmissivity τ and then try to find the maximum fidelity as in the proof of Theorem 3. We have that the two diagonal elements of the noise matrix N now read

$$\alpha(\tau) = \beta(\tau) = n - 2\tau d + \tau^2 m + 1 - \tau^2, \tag{53}$$

The optimal map must minimize $\det N = \alpha^2$, and therefore we impose

$$\frac{d\alpha(\tau)}{d\tau} = 0, \tag{54}$$

which is satisfied iff $\tau = d/(m-1)$. If $0 < \tau < 1$, this map composed by the local symplectic map and the attenuation is the optimal map. If instead for any $\tau \in [0, 1]$, the condition of Eq. (52) is not satisfied, there is no critical point in this interval and therefore the optimal map is just the symplectic transformation to the normal form V_1. One has also to check the behavior at the lower boundary value $\tau = 0$, but this is trivial because this means that Bob uses the vacuum to implement the teleportation which is never the optimal solution if we have an entangled channel. In fact, if Bob uses the vacuum, the channel looses its quantum nature and the maximum of the fidelity is the classical one $\mathcal{F} = 1/2$, which is below the lower bound for any entangled state given by Eq. (12). □

Theorem 5 therefore characterizes in detail the optimal TGCP map, giving in particular the conditions under which this map is noisy, i.e., non-symplectic and therefore when teleportation is improved by *increasing the noise and decreasing the entanglement* of the shared state.

Using this latter theorem we can also determine how, from an operational point of view, one can compute

the value of the teleportation fidelity maximized over all TGCP maps, starting from the symplectic invariants of the bipartite Gaussian state initially shared by Alice and Bob. From the CM of this latter state one can:

1. compute the four symplectic invariants $a = \sqrt{\det A}$, $b = \sqrt{\det B}$, $c = \sqrt{|\det C|}$ and $v = \det V$.

2. Knowing the first three invariants of the channel, the elements n, m, and d of the normal form V_η can be expressed as functions of only the two unknown parameters λ and η as

$$n(\lambda) = -\lambda/2 + \sqrt{a^2 + (\lambda/2)^2}, \tag{55}$$

$$m(\lambda, \eta) = -\lambda/2\eta^2 + \sqrt{b^2 + (\lambda/2\eta^2)^2}, \tag{56}$$

$$d(\lambda, \eta) = -\lambda/2\eta + \sqrt{c^2 + (\lambda/2\eta)^2}. \tag{57}$$

3. The two parameters λ and η can be found solving the following system in the region $0 < \eta < 1$,

$$\begin{cases} \det V_\eta(\lambda, \eta) = v \\ \eta\,[m(\lambda, \eta) - 1] = d(\lambda, \eta) \end{cases} \tag{58}$$

and solving also the first equation in the boundary $\eta = 1$,

$$\det V_1(\lambda) = v, \tag{59}$$

(the two conditions of (58) come from the invariance property of the determinant of the channel and form the maximization condition of Eq. (52)).

4. We call (λ_i, η_i) with $i = 1, 2 \ldots k$, the union of the solutions of (58) and (59) (we have at least one solution because (59) admits at least a solution). We then compute the candidate fidelities

$$F_i = 2\left[2 + \sqrt{a^2 + \lambda_i^2/4} + \sqrt{b^2\eta_i^4 + \lambda_i^2/4} - 2\sqrt{c^2\eta_i^2 + \lambda_i^2/4} + 1 - \eta_i^2\right]^{-1}, \tag{60}$$

so that the maximum fidelity will be $\mathcal{F}_{opt} = \max\{F_i\}$.

V. CONCLUSIONS

We have studied the Braunstein-Kimble protocol [3] for the CV teleportation of coherent states and how the corresponding fidelity can be maximized over local operations at Alice and Bob site. We have assumed that Alice and Bob share a Gaussian bipartite state and restricted to Gaussian LOCC strategies, which preserves such a Gaussian property. We have shown that, for a given shared entanglement, the maximum fidelity of teleportation is bounded below and above by simple expressions depending upon the lowest PT symplectic eigenvalue ν only (see Eq. (1)). We have seen that these bounds are quite tight and that the upper bound of the fidelity is reached if and only if Alice and Bob share a symmetric entangled state. We have also determined the general form of the CM of Alice and Bob state after the optimization procedure. Then we have restricted to local TGCP maps and shown that the optimal TGCP map is composed by a local symplectic map, eventually followed by an attenuation either on Alice or on Bob mode. Finally we have shown how the corresponding value of the maximum fidelity $\mathcal{F}_{opt}$ can be derived from the knowledge of the symplectic invariants of the initial CV entangled state shared by Alice and Bob.

If one considers generic GCP maps (i.e., including also Gaussian measurements on ancillas), one expects to further improve the teleportation; in this case one should adopt the description of GCP maps given in [24] in order to characterize the optimal local Gaussian map, but this will be the subject of future work.

Another open question is to see if teleportation fidelity can be increased by leaving the Gaussian setting studied here and consider more general non-Gaussian local operations at Alice and Bob site. In this respect, the preliminary results of Refs. [25] seem promising.

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VII. APPENDIX

We now prove the “only if” part of Corollary 1, i.e., that if $\mathcal{F}_{opt} = 1/(1+\nu)$ (the upper bound of the optimal fidelity), then the bipartite Gaussian state shared by Alice and Bob is symmetric.

Proof. Theorem 3 shows that the final CM after any optimization map must be V_1 of Eq. (29). Using Lemma 1 and the explicit form of V_1, the hypothesis is equivalent to $n + m - 2d = 2\nu$, so we can make the substitution $d = (n+m)/2 - \nu$, which is just a different parametrization: $V_1(n, m, d, \lambda) \to V_1(n, m, \nu, \lambda)$. Now, the condition that ν is equal to the PT minimum symplectic eigenvalue gives us a constraint on the parameters of the matrix V_1

$$\nu\{V_1(n, m, \nu, \lambda)\} = \nu. \tag{61}$$

If the state is symmetric, which means $n = m$, then the condition of Eq. (61) is identically satisfied. If the state is non symmetric, Eq. (61) is a non-trivial equation that solved for λ gives $\bar{\lambda} = (m-n)^2/8\nu - n - m$. However the corresponding matrix $V_1 = (n, m, \nu, \bar{\lambda})$ is not the CM of a physical state; in fact, the characteristic polynomial of V_1 can be written as $P(x) = (c_0 + c_1 x + x^2)(g_0 + g_1 x + x^2)$ where $c_0 = -\nu(n + m + \nu)$, but this means that V_1 has at least one negative eigenvalue and therefore it is not

positive definite. Therefore $\mathcal{F}_{opt} = 1/(1+\nu)$ is realized only if Alice and Bob state is a Gaussian symmetric state. $\square$

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